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Two-component universe containing ordinary mater and a new component of the form $\frac{l}{R}\rho$

V. Majerník
Institute of Mathematics Slovak Academy of Sciences,
SK-814 73 Bratislava, Štefánikova 49, Slovak Republic

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Abstract

We consider a model of the universe described by Einstein equations whose the right-hand side consists of ordinary energy-momentum tensor and an effective vacuum energy-momentum tensor with $\rho^{vac} = -p^{vac} = L_c/K$, where $L_c = 8\pi Gl\rho R^{-1}$ and ρ is the mass density of cosmical medium, l the constat with dimension $[cm]$ and R the scale factor of the expanding universe. We determine the vacuum energy density assigned to the term L_c as a function of the scale factor and, when taking $l = 2R_0$, R_0 being the present 'radius' of the universe, and $\Omega_0 = 0.3$, the normalized vacuum mass density, we obtain $\Omega_t = 1$, deceleration parameter $q_0 \approx -0.5$ and the age of the universe $t_0 \approx 0.9 \times H_0^{-1}$. The values are compatible with the estimated observatory values of cosmic parameters. Finally, we try to interpret the energy content of L_c as the field energy of a field whose source is the ordinary matter in the universe.

KEYWORDS: Cosmology; decaying cosmological constant; dark energy; cosmic expansion.
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1 Introduction

Since the discovery of the accelerating expansion of the universe [1], dark energy is often invoked to explain this phenomena. The basic set of astronomical observations confirming the existence of dark energy includes: observations from SNe Ia, CMB anisotropies, large scale structure, X-ray data from galaxy clusters, age estimates of globular clusters and old high

red-shift galaxies (OHRG's). These observatory results seem to supply the remaining piece of information confirming the inflationary flatness prediction ($\Omega_t \approx 1$). The existence of an extra component filling the universe has also indirectly been suggested by independent studies based on fluctuations of the 3K relict radiation [10], large scale structure [2] age estimates of globular clusters or old high redshift objects [3], as well as by the X-ray data from galaxy clusters [4]. These observations strongly suggest that the universe is flat composed of $\rho \sim 1/3$ of matter (baryonic + dark) and $\rho_\lambda \sim 2/3$ of an exotic component.

In the framework of quantum field theory the presence of dark energy is due to the zero-point energy of all particles and fields filling the universe which manifests itself in several quantum phenomena like the Lamb shift and Casimir effect [11]. However, the conflict between the estimating observatory value of dark energy and that of quantum field prediction [9] inspired many authors to find alternative for the field-theory model of dark energy. The well-known alternative is the Einstein cosmological constant Λ which is a time independent and spatially uniform dark component. It may be classically interpreted as a relativistic perfect fluid obeying the equation of state $p = -\rho$. Recently, many models with variable cosmological constant have been proposed, in some cases it depends on scale factor [6],[12] or on the cosmic time [7] or on both of them (for an overview see e.g. [13]). Other authors have suggested that the cosmological constant can be written as a trace of an energy-momentum tensor [8].

In this report, we study the cosmic evolution supposing that the energy-momentum tensor in Einstein's equations consists of ordinary energy-momentum tensor and an effective vacuum energy-momentum tensor with $\rho^{vac} = -p^{vac} = L_c/K$, where $L_c = 8\pi Gl\rho R^{-1}$ and ρ is the mass density of cosmical medium, one $L_c = 8\pi l\rho R^{-1}$, where ρ is the mass density of the cosmic medium R is the scale factor and l is a free constant having the length dimension.

2 Investigate of cosmic evolution described by the modified energy-momentum tensor

We start with the Einstein equations

$$R^{ik} - (1/2)R = -KT^{ik} + L_c g^{ik} \qquad K = 8\pi G.$$

L_c can be phenomenologically viewed as a dynamical cosmological term. Using the Robertson-Walker metric for flat space ($k = 0$) and $L_c = Kl\rho R^{-1}$,

we have

$$(\frac{\dot{R}}{R})^2 = \frac{K}{3}(\frac{l\rho}{R}) + (\frac{K}{3})\rho = (\frac{K\rho}{3})(1+\frac{l}{R}) \tag{1}$$

and

$$\frac{2\ddot{R}}{R} + (\frac{\dot{R}}{R})^2 = -Kp + \frac{Kl\rho}{R}. \tag{2}$$

The corresponding total energy-conservation law reads [12]

$$\frac{d}{dR}(\rho R^3) + 3pR^2 = -\frac{1}{K}[\frac{d}{dR}(L_c R^3) - 3L_c R^2] = -\frac{1}{K}[\frac{d}{dR}(K\rho l R^2) - 3Kl\rho R]. \tag{3}$$

One can view $-(L_c/K)g^{ik}$ as the effective vacuum energy-momentum tensor with $\rho^{vac} = -p^{vac} = L_c/K$. The vacuum energy density assigned to L_c is

$$\rho_c = (\frac{l}{R})\rho. \tag{4}$$

For the radiation-dominated epoch ($p = 1/3$), equation(3) leads to the following differential equation

$$\frac{d\rho_r(R)}{dR}(R^3 + lR^2) + \rho_r(R)(3R^2 + R^2 - lR) = 0. \tag{5}$$

Its solution is

$$\rho_r(R) = \frac{C_1 R}{(l+R)^5}, \tag{6}$$

where C_1 is a positive integration constant. For the mater-dominated epoch ($p = 0$), equation(5) gets the form

$$\frac{d\rho_m(R)}{dR}(R^3 + lR^2) + \rho_m(R)(3R^2 - lR) = 0, \tag{7}$$

whose solution is

$$\rho_m(R) = \frac{C_2 R}{(l+R)^4}, \tag{8}$$

where C_2 being again a positive integration constant. The solutions of equation(6) and equation(8), $\rho_r(R)$ and $\rho_m(R)$, as functions of R are shown in Fig. 1.

One sees that their graphical representations show one-hump curves whose maxima lie in $R = l/3$ for (p=0) and in $R = l/4$ for (p=1/3), respectively.

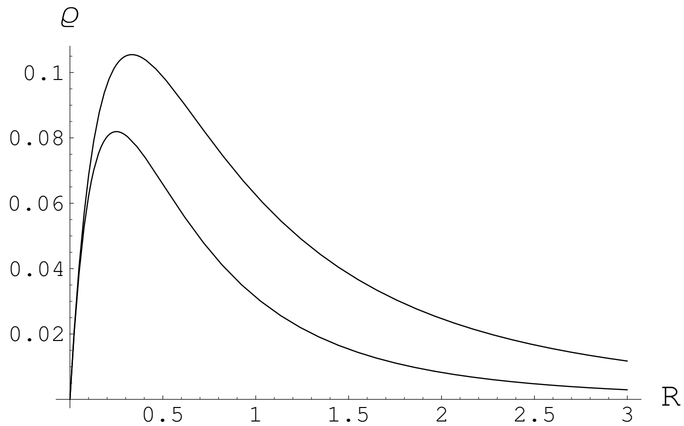

Figure 1: ρ as a function of R, $(l = 1)$. The upper curve refers to ρ_m and the lower to ρ_r.

Knowing ρ as a function of R we can determine $R(t)$ for all time by solving equation(1) and equation(2). Substituting $\rho_r(R)$ or $\rho_m(R)$ into equations (1) and (2) we get for $p = 1/3$ or $p = 0$

$$(\frac{\dot{R}}{R})^2 = \frac{K}{3}(\frac{C_1 l}{(l+R)^5}) + \frac{K}{3}(\frac{C_1 R}{(l+R)^5}) = \frac{K}{3}(\frac{C_1}{(l+R)^4}). \tag{9}$$

and

$$\frac{2\ddot{R}}{R} + (\frac{\dot{R}}{R})^2 = K((\frac{C_1 R}{(l+R)^5}) - p) \tag{10}$$

or

$$(\frac{\dot{R}}{R})^2 = \frac{K}{3}(\frac{C_2}{(l+R)^3}) \tag{11}$$

and

$$\frac{2\ddot{R}}{R} + (\frac{\dot{R}}{R})^2 = K(\frac{C_2 R}{(l+R)^4}), \tag{12}$$

respectively. equation (9) can be rewritten in the form

$$\dot{R} = \sqrt{\frac{KC_1}{3}}(\frac{R}{(l+R)^2}). \tag{13}$$

If $l \gg R$, then the right-hand side of equation(13) reduces to the form

$$(\frac{\dot{R}}{R})^2 \approx (\frac{K}{3})(\frac{C_1}{l^4}).$$

Its solution represents an exponential function

$$R(t) \approx \exp \frac{\sqrt{C_1 k} t}{\sqrt{3} l^2},$$

i.e. in the initial period R grows exponentially resembling the extremely rapid expansion of the universe during its inflationary period. On the other side, if $l \ll R$ then we have

$$(\frac{\dot{R}}{R})^2 \approx \frac{K}{3} \frac{C_1}{R^4}.$$

The solution of this equation represents the time dependence of scale factor, well-known in the standard cosmology

$$R(t) = \sqrt{\frac{2KC_1 t}{3}}.$$

Dividing equation(1) by square of the present Hubble constant H_0^2 we have

$$1 = \Omega_0 + \Omega_\Lambda = \Omega_0(1 + \frac{l}{R_0}) = \Omega_t. \tag{14}$$

Here

$$H_0^2 = (\frac{\dot{R}_0}{R_0})^2, \qquad R_0 = 1, \qquad \Omega_0 = \frac{\rho_0}{\rho_c}, \qquad \Omega_\Lambda = \frac{L_c}{3H_0^2} = \Omega_0(\frac{l}{R_0}) \qquad \rho_c = \frac{3H_0^2}{8\pi G}.$$

ρ_c is the critical mass density.

Subtracting equation (2) from equation(1) we get

$$\frac{\ddot{R}}{R} = -\frac{8\pi}{3} G\rho + \frac{2}{3} L_c = -\frac{K\rho}{3}(1 - \frac{2l}{R}). \tag{15}$$

By means of equation (12) and equation(13) we obtain the deceleration parameter q_0 as a function of Ω_0 and Ω_Λ in the form [14]

$$q_0 \equiv -\frac{\ddot{R}R^2}{\dot{R}^2 R} = \frac{\Omega_0}{2} - \Omega_\Lambda = \Omega_0(\frac{1}{2} - \frac{l}{R_0}). \tag{16}$$

In our model, the expression of the cosmic age t_0 in terms of Ω and Ω_Λ is determined by the integral

$$t_0 = H_0^{-1} \int_0^1 \frac{dR}{R} [\frac{\Omega_0}{R^3} + \frac{\Omega_0 l}{R_0}]^{-1/2}. \tag{17}$$

The result of the integration of the expression for t_0, as a function of Ω_0 and R_0, has the form

$$t_0 = H_0^{-1} \frac{2\arcsin[\frac{l}{R_0}]}{3\sqrt{\Omega_0 l/R_0}}. \tag{18}$$

Given Ω_0, t_0 becomes only a function of l. Our time-depending L_c predicts creation of matter at present with an rate of creation per unit volume given by $(C_2 \approx M_t)$

$$RC = \frac{1}{R^3}\frac{d(\rho_m R^3)}{dt}\mid_0 = \frac{4lC_2R_0H_0}{(l+R_0)^5} = \frac{4l\rho_m H_0}{l+R_0}. \tag{19}$$

3 Consequences

(i) The key role in our further consideration plays the value of l. There are basically two important characteristic lengths in cosmology which can be substituted for l, (i) Planck's length $l_P = \sqrt{\frac{hG}{c^3}} \approx (10^{-37}cm)$ and (ii) the length assigned to the total mass of the universe $l_c = \frac{GM_t}{c^2} \approx 2R_0$, where R_0 is the present radius of the universe $R_0 \approx 10^{28}cm$. Taking $l = 2R_0$ and $\Omega_0 = 0.3$ we get, when inserting this l into equation(14), equation(16), equation(18) and equation(19) the following values for the corresponding quantities $\Omega_T \approx 1$, $t_0 \approx 0.98 \times H_0^{-1}$, $q_0 \approx -0.5$ and $CR = 10^{-47}g^1cm^{-3}s^{-1}$. The later value is about seven order lower than the creation rate in the steady state theory. These values are compatible with the estimated observatory ones.

(ii) As shown in Fig.(1), if $R = 0$ then ρ_r and ρ_m become zero. If $R \gg l$ then $\rho_m(R) \propto C_1R^{-3}$ and $\rho_r(R) \propto C_2R^{-4}$,respectively. The maximum value for ρ_m and ρ_m lies in $l/4$ and $l/3$, respectively.

(iii) The acceleration of cosmic evolution continues until the right-hand side of equation(15) remains positive then begins its deceleration.

(iv) The fate of the universe in our model is different than the fate nowadays believed. In a flat or open universe without dark energy, the cosmic expansion continues forever, and the horizon grows more rapidly than the scale factor. In our model vacuum energy represents a dynamical variable which initially drives the expansion of the universe, later evolves to the present-day small value and in future it becomes continuously smaller converting to zero.

(v) Since the mass density assigned to $L_c \propto l_c\rho R^{-1}$ is a dynamical quantity we will determine its mean value over the whole volume of the universe. For

the total amount of the vacuum energy in the universe, we have

$$M_c = \int\limits_0^{R_0} (\frac{l_c\rho}{R}) 4\pi R^2 dR \approx l_c \rho R_0^2 \tag{20}$$

Hence, its mean value is $\rho_c' \approx M_c/R_0^3 = l_c\rho R_0^{-1}$. With $l_c \approx 2R_0$ we have $\rho_c' \approx 2\rho$. Recent astronomical observation indicates that the density of the dark energy is small positive and approximately equal to mass density in the universe.
(iv) If one takes $l = l_P = \frac{Gm_P}{c^2}$,where m_P is Planck mass $m_P = \sqrt{\frac{hc}{G}}$ then one has again $\rho_P(t=0) = 0$. However, in this case R grows extremely rapid just on very beginning of cosmic evolution resembling the cosmic inflation. Later, it quickly starts to evolve very closely to the standard model, so that it has no influence on the present-day universe. The above-considered universe is two-component consisting of the ordinary matter ρ and the energy assigned to L_c. To have the accelerating universe, when taking $l = l_P$, we need an additional component causing its present acceleration. The simplest way is to add a constant cosmological term Λ to L_c. The addition of Λ to L_c does not change the solutions of equation(4) and equation(7). It changes the equations (1) and (2) in that Λ is added to their right-hand sites . Such three-component universe is accelerating, without initial singularity and with an inflation phase on its beginning.

Finally, we will point to the interesting fact that the formula for the total amount of the vacuum energy in the universe given by equation(20) and that calculated classically within the gravito-dynamical field theory (analog to electrodynamics) [16] are similar. In the gravito-dynamic theory, the analog to electric charge or charge density is the expression $\sqrt{G}m$ or $\sqrt{G}\rho$ [15], respectively. Hence, the intensity of gravitational field in a sphere with spherically uniform mass distribution whose radius is R is given by the well-known equation

$$\bigtriangledown I_g = -4\pi\sqrt{G}\rho,$$

where ρ is the mass density in the sphere. Its solution has the form

$$I_g = -\frac{4\pi}{3}\sqrt{G}\rho R$$

Likewise, as in the case of the electromagnetic field, we determine the field energy density of gravitational field E_g as

$$E_g = \frac{1}{8\pi}(I_g)^2 = \kappa G\rho^2 R^2 \qquad \kappa = (\frac{4\pi}{3})^2(\frac{1}{8\pi}) \approx 1.$$

If we model the universe as a sphere filled by the uniform mass density ρ having radius R_0 we get for the content of the field energy-mass

$$M_f = \int_0^{R_0} (\frac{G\rho^2 R^2}{c^2}) 4\pi R^2 dR \approx G\rho^2 c^{-2} R_0^5 \approx \frac{GM}{c^2}\rho R_0^2 = l_c \rho R_0^2. \qquad (21)$$

We see that the formulas for M_c and M_f are similar. This suggests that the dark energy might represent the field energy whose source is the mass content of the universe. The field energy can be included into the Einstein equations through the term L_c.

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